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Dynamics and Multivariable Control Analysis for Anaerobic Digestion

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ABSTRACT

The stabilization of wastewater sludge by anaerobic digestion has been analysed by developing a multivariable control strategy. The dynamic model of the process was solved by digital simulation. The process variables have been classified into controlled, manipulative, disturbance and inventory variables. The dynamic model was linearized into a state space model, from which a frequency response model was obtained. This model was analysed in terms of interaction of variables and integral controllability. A control system has been developed which manipulates heat addition and inflow so as to control digestion temperature and total organic carbon respectively. It has been found that the stability of the system and its integrals are controllable. Furthermore the control system satisfies the major objectives of digestion, namely sludge stabilization and steady biogas production. The dynamic characteristics of thermophilic digestion are compared with those of mesophilic operation. While the stability and interaction properties were found to be comparable for the two processes, thermophilic operation has a shorter recovery time after upsets.

1. INTRODUCTION

Anaerobic digestion is one of the most widely used sludge stabilization processes for both domestic and industrial wastes. The process (Fig. 1) is based mainly on biological activity, *i.e.* the biodegradation of organic substrates in order to improve dewaterability, to pasteurize the sludge and to produce an energy-rich biogas.

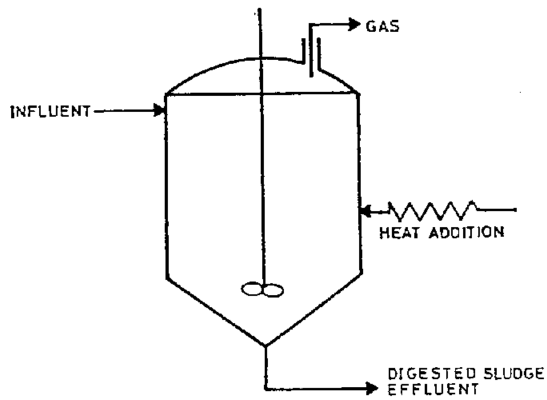

Fig. 1. The anaerobic digester.

Because of the complexity of energy problems around the world, coupled with increasing health awareness in urban areas, major attention is being given to improvements in the performance of this energy-efficient process and it has been noted by many authors that a major area of improvement would be the use of automation to maintain specific quality and operating objectives [1, 2].

A number of publications have appeared proposing automatic control strategies for anaerobic digestion. For example, a control system has been proposed [3, 4] which maintains the optimum pH value in the digester, and Maeda [5] has described a multi-

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layer control strategy for determining optimal values for temperature and inflow rate.

The purpose of the present work was to investigate the interaction and dynamic properties of anaerobic digestion. Through appropriate selection of the control objectives and method of process manipulation, a multivariable control system is proposed. The possibility of applying a multivariable strategy is examined using input–output type models, generated from a non-linear process model. Specifically, the questions of interaction and stability have been addressed and a multivariable control strategy developed. It has been assumed throughout that domestic sludge is treated under typical operating conditions as described in the literature. Thermophilic digestion is compared with the mesophilic process as regards dynamic and steady state characteristics.

2. PROCESS CONTROL PARAMETERS

2.1. Temperature

Anaerobic digestion can be carried out over the temperature range 20 - 60 °C. Mesophilic digestion commonly occurs around 30 - 35 °C and thermophilic digestion around 50 - 60 °C. It is well known that solid retention time decreases at higher temperatures requiring heat input to the digester. Because of the pasteurization effect which occurs at the higher temperatures, thermophilic digestion can eliminate the need for further pasteurization and thus it has a potential advantage over mesophilic operation. Furthermore, smaller vessels can be used because of the shorter retention time. However, the operating space is limited at higher temperatures and tight control over temperature is needed to guarantee an optimum growth rate [6]. This limitation on operating space calls for the use of modern control analysis and better appreciation of the interaction of the temperature with the other variables.

2.2. Solids retention time (SRT)

SRT is the time that an average solid particle stays within the digester before it is drawn into the outflow. In a well-stirred reactor the solids are drawn along together with the outflow, and then the SRT is equal to the hydraulic retention time (HRT). The SRT is affected by the digestion temperature, by the wastewater concentration and by the type of microorganisms:

$$\mathrm{SRT} = \mathrm{HRT} = \frac{V}{Q} \tag{1}$$

where V is the volume of the digester contents and Q is the inflow rate. Thus if V is kept constant, the SRT can be changed by changing Q. It should be noted, however, that the SRT is a control objective mainly for its direct effect on stabilization efficiency.

2.3. pH

pH values around the neutral range (6.8 - 8) are considered optimum for the growth of methanogens. Many domestic wastewaters are within this range and hence neutralization is not usually needed. Acidic contaminants are occasionally present owing to discharge from neighbouring factories and metal works. Thus periodic monitoring is advisable to ensure the desired pH value. Some sewage treatment plants adjust their wastewater pH at the inlet to the primary treatment, thus eliminating the need for further neutralization.

2.4. Environmental disturbances

Variations in inlet temperature T_0 are to be expected between day and night operations and long-term temperature variations occur between the winter and summer seasons. These changes may exceed 10 °C in the Kuwaiti climate.

Wastewater concentrations of organic matter S_0 typically change after meal times. Peak concentrations are around 2 p.m. and 9 p.m. Organic shock loads occur for various reasons. In some farming neighbourhoods extreme shock conditions may develop through random or periodic washing. Wastewater flow also varies between day and night. These short-term variations may cause significant problems for treatment units. Long-term variations in wastewater flow occur, especially during the summer months. The exodus during vacations may cause wastewater flows to drop and some units to run at reduced capacity.

Oxidants such as nitrates, sulphates and H_2O_2 are known to inhibit methanogenesis. Toxic matter which affects biological reactions should not be present persistently in domestic wastewater. Since oxidants and toxicants are difficult to track continuously, any control system should be able to monitor product quality and make adjustments to regulate these and other temporary disturbances.

3. MODELLING AND SIMULATION

Accurate modelling of anaerobic digestion is difficult owing to the complexity of the reactions and the hydrodynamics of the process. Because of the evolution of gas and the presence of solids, anaerobic digestion is essentially a three-phase process. For control system studies, a simplified model such as the single microbial growth equation proposed by Monod [7] may be adequate:

$$\mu = \frac{\mu_{max} S}{K + S} \tag{2}$$

The endogenous decay coefficient is neglected since it has been shown to make a relatively small contribution to the overall digestion kinetics [8]. The maximum specific growth rate is given [5] as a function of temperature T, according to the relation

$$\mu_{max} = 0.013T - 0.129 \tag{3}$$

This equation is valid for temperatures between 20 and 60 °C. However, other forms of temperature dependence have been proposed based on the Arrhenius equation [8].

The digester is assumed to have stirred tank continuous flow operation. Hence temperature, substrate concentration(s) and micro-organism concentration X are assumed to be uniform and to have the same values at the digester outlet. This assumption is valid for many digester designs where mixing is achieved mechanically or through biogas recirculation. The dynamic balance equations can be written as

$$\frac{dX}{dt} = \mu X - \frac{Q}{V} X \tag{4}$$

$$\frac{dS}{dt} = \frac{Q}{V}(S_0 - S) - \frac{\mu X}{Y_S} \tag{5}$$

$$\rho V Cp \frac{dT}{dt} = \rho Q\, Cp(T_0 - T) + E \tag{6}$$

where E is the energy input in J day^{-1}.

Equation (6) can be simplified as

$$\frac{dT}{dt} = \frac{Q}{V}(T_0 - T) + G_u \tag{7}$$

where

$$G_u = E/\rho V Cp \tag{8}$$

Gas production can be expressed in terms of the growth rate and a gas yield coefficient Y_g:

$$G = Q Y_g \mu X \tag{9}$$

More sophisticated models than this have been proposed [3], which differentiate between the acidogenic phase and the methanogenic phase of digestion. However, the simplified overall model described by eqns. (4) - (8) has been found to be adequate [5], and lends itself more easily to linear analysis. The operating conditions for thermophilic and mesophilic digestion are given in Table 1. Equations (2) - (7) were solved by digital simulation, and the effect of some variables was studied using step tests. The results are plotted in Figs. 2 - 7 for changes in G_u, Q, T_0 and S_0. From these figures, it can be seen that the dominant time constants for the process are 10 and 30 days for the thermophilic and mesophilic processes respectively. The effect of a disturbance in S_0 on S gives a particular shape to the graph. The outflow concentration peaks momentarily in response to a sudden increase in S; then it decreases because of a build-up of the micro-organism concentration X which consumes the excess organic load. The rate of this organic consumption depends largely on the

TABLE 1

Operating conditions for anaerobic digestion

	Thermophilic (T)	*Mesophilic (M)*
Q (m^3 day^{-1})	300	100
V (m^3)	3000	
S_0 (mg COD l^{-1})	9000	
T_0 (°C)	30	
T (°C)	55	35
S (mg COD l^{-1})	300	
X (m^3 vss m^{-3})	0.5	
K (mg l^{-1})	1458	678
Y_s (m^3 vss m^{-3} (mg COD l^{-1})$^{-1}$)	5.75×10^{-5}	1.724×10^{-5}
Y_g (m^3 methane (m^3 vss)$^{-1}$ day^{-1})	3.045	1.015

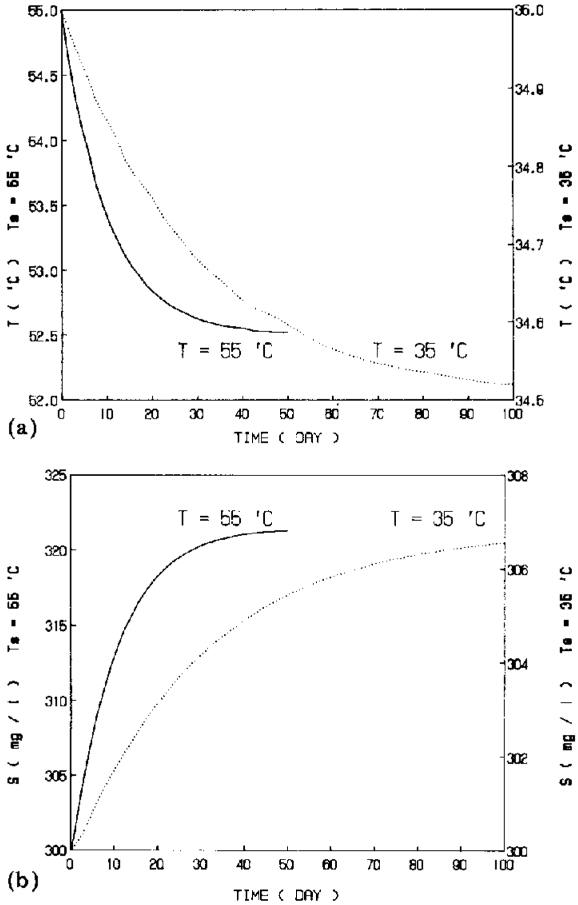

Fig. 2. (a) −10% step change in G_u (heat injection): effect on temperature; (b) −10% step change in G_u (heat injection): effect on bioconversion.

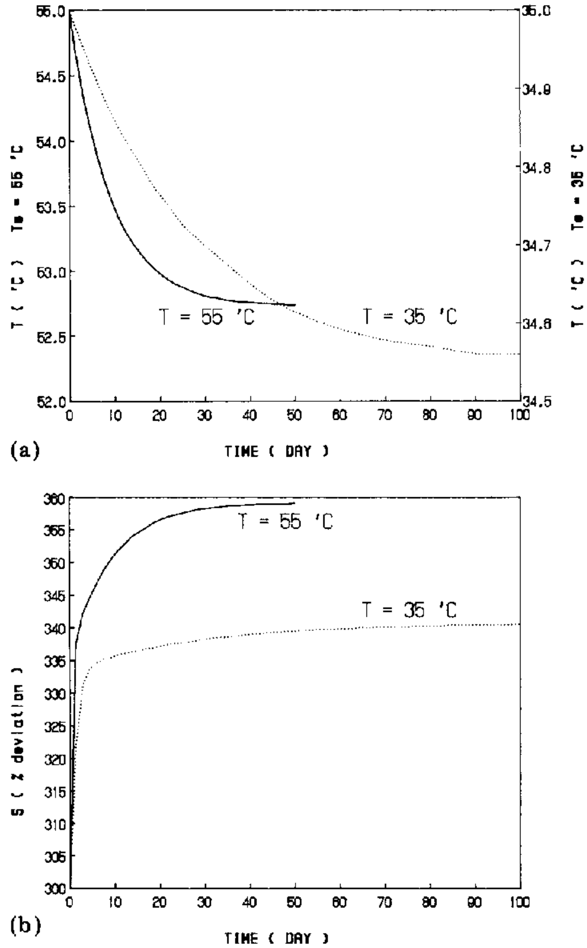

Fig. 3. (a) +10% step change in Q (inflow rate): effect on temperature; (b) +10% step change in Q (inflow rate): effect on bioconversion.

Monod kinetic parameters and hence on the temperature.

The above response shows that the digestion kinetics has a capacity for self-regulation in the face of organic shock loads. The responses of S to Q and G_u show that both variables have a marked effect on effluent concentration, albeit in different directions.

4. MULTIVARIABLE CONTROL ANALYSIS

From the considerations discussed above different sets of quality variables can be identified. The digestion temperature should be held constant, and this is easily measured. Thus T defines one controlled objective Y_1. The degree of stabilization S is an operational and environmental objective and hence should be maintained at the allowed level. This sets the second controlled objective Y_2. The only problem is that a continuous and fast measurement of stabilization efficiency must be available to allow effective automatic control.

Measurement of the biochemical oxygen demand (BOD) is not suitable since it takes a long time to prepare (5 days). Chemical oxygen demand (COD) takes three hours which is still too long. Measurement of the total organic carbon (TOC) has gained considerable support now that fast and reliable devices have been developed. Roy *et al.* [9] described one such device which depends on a UV-promoted chemical oxidation technique. A TOC measurement can be obtained in 3 - 7 min which is short enough compared with the frequency of disturbances and the digester dynamics. Thus stabilization based on TOC is taken as Y_2. The process or manipulative variables are obvious. Heat addition G_u has a direct effect

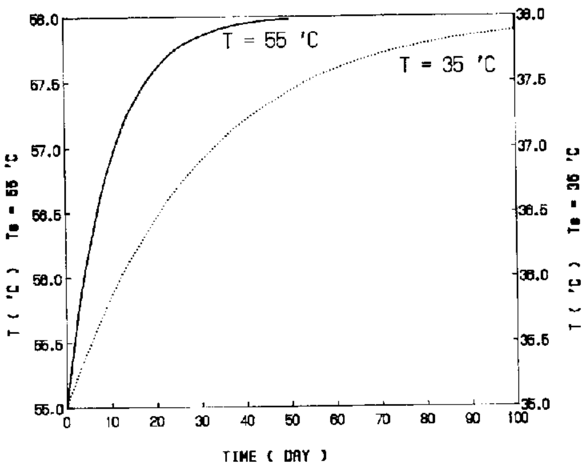

Fig. 4. +10% step change in T_0 (inlet temperature): effect on temperature.

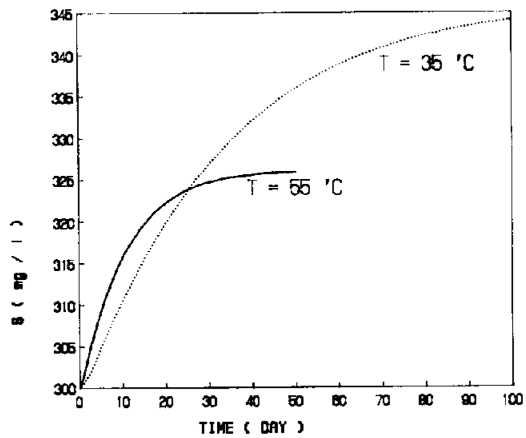

Fig. 5. —10% step change in T_0 (inlet temperature): effect on bioconversion.

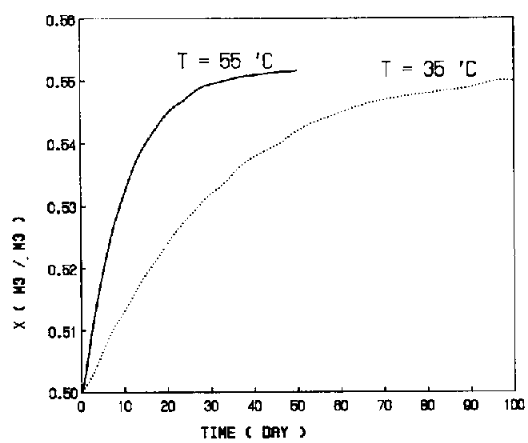

Fig. 6. +10% step change in S_0 (inlet concentration): effect on biomass.

on both T and S. In practice, G_u can be applied by circulating some of the digester contents through a heat exchanger. The

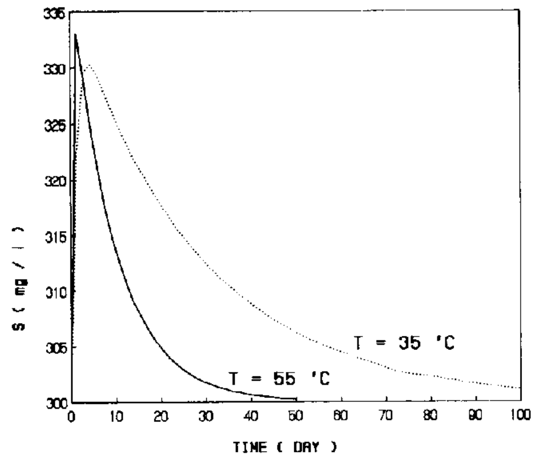

Fig. 7. +10% step change in S_0 (inlet concentration): effect on bioconversion.

method of heating can be external or by burning some of the digester gas to maintain energy integration. The inflow rate Q can be handled as the second process input, though to make this more effective, the digester liquid inventory (or level) should be controlled, *e.g.* by a float device. Thus the SRT or HRT can be controlled simply by changing Q as necessary. The gas inventory (pressure) can be controlled by a control valve in the gas exit line.

5. LINEAR METHODS

In order to obtain a linear model for the digester, the non-linear model (eqns. (2) - (7)) must be linearized. The linearized equations are given in Appendix A. Arranging the equations in the state space form:

$$\dot{X} = \mathrm{A}X + \mathrm{B}U + \Gamma d \tag{10}$$

$$Y = \mathrm{C}X \tag{11}$$

while the vectors X, U, d and Y take the form

$$X = \begin{bmatrix} S \\ T \end{bmatrix} \qquad U = \begin{bmatrix} Q \\ G_u \end{bmatrix}$$

$$d = \begin{bmatrix} T_0 \\ S_0 \end{bmatrix} \qquad Y = \begin{bmatrix} \mathrm{TOC} \\ T \end{bmatrix}$$

From the state space model, a frequency input–output model can be developed from the transformation

TABLE 2

Transfer matrices for an anaerobic digester

$G_{p11} = \dfrac{\{1/(F7*F6)\}*\{(S+F6)*(F11*F5)+F11*F9*F10\}}{\{(1/F7*S-1)(1/F6*S+1)\}}$	$F1 = \dfrac{0.013*S_s*X_s}{(K+S_s)}$
$G_{p12} = \dfrac{\{(F11*F9)/(F7*F6)\}}{\{(1.0/F7*S-1)*(1.0/F6*S+1)\}}$	$F2 = \dfrac{(0.013*T_s-0.129)*S_s}{\{(K+S_s)-Q_s/V\}}$
$G_{p21} = \dfrac{(F10/F6)}{(1.0/F6*S+1)}$	$F3 = \dfrac{(0.013*T_s-0.129)*X_s*K}{(K+S_s)^2}$
$G_{p22} = \dfrac{(1.0/F6)}{(1.0/F6*S+1)}$	$F4 = \dfrac{-X_s}{V}$
$G_{d11} = \dfrac{F6*F11*F9}{\{(S-F7)*(S+F6)\}}$	$F5 = \dfrac{(S_{os}-S_s)}{V}$
$G_{d12} = \dfrac{F11*F6}{(S-F7)}$	$F6 = \dfrac{Q_s}{V}$
$G_{d21} = \dfrac{F6}{(S+F6)}$	$F7 = \dfrac{-(0.013*T_s-0.129)*K*X_s}{\{YX*(K+S_s)^2-Q_s/V\}}$
$G_{d22} = 0.0$	$F8 = \dfrac{-(0.013*T_s-0.129)*S_s}{\{YX*(K+S_s)-Q_s/V\}}$
	$F9 = \dfrac{-0.013*S_s*X_s}{\{YX*(K+S_s)\}}$
	$F10 = \dfrac{(T_{os}-T_s)}{V}$
	$F11 = \dfrac{1.0}{3.0}$

$$\mathbf{G}_p = \mathbf{C}(S\mathbf{I}-\mathbf{A})^{-1}\mathbf{B} \tag{12}$$

$$\mathbf{G}_d = \mathbf{C}(S\mathbf{I}-\mathbf{A})^{-1}\Gamma \tag{13}$$

$$Y = \mathbf{G}_p U + \mathbf{G}_d d \tag{14}$$

The transfer matrices $\mathbf{G}_p$ and $\mathbf{G}_d$ are listed in Table 2.

From the matrices $\mathbf{G}_p$ and $\mathbf{G}_d$ one can visualize that the overall process dynamics are governed by a dominant time constant which is equivalent to the HRT. For the thermophilic case, the HRT is 10 days, which is around one third of that needed for equivalent stabilization at mesophilic temperatures. Thus thermophilic digestion leads to faster dynamics and hence a faster control action can be utilized. The relative speed of response can be seen clearly in Figs. 2 - 7. It can be concluded that thermophilic operation has a faster rate of self-regulation than mesophilic operation. In either case, the time constant is large enough so that the problem of controller tuning is not very critical. The fact that the matrix $\mathbf{G}_p$ has non-zero entries means that there is considerable interaction between the input and output variables. Understanding these interaction properties can contribute greatly to the design of stable and effective control strategies.

6. INTERACTION ANALYSIS

The block diagram in Fig. 8 represents the relation between the inputs Q and G_u,

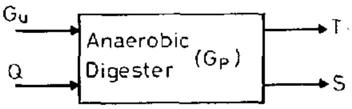

Fig. 8. Block diagram for anaerobic digester operation.

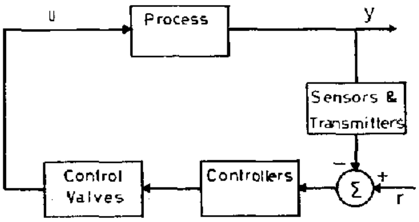

Fig. 9. Feedback control of a multivariable process.

disturbances T_0 and S_0, and the outputs T and S when no control is enforced.

The multivariable control system takes the form of Fig. 9. A major problem is to design the controller matrix **K** so that the controller variables Y are kept around their specified values (set points) $\boldsymbol{r}$. The control algorithm should move the inputs U through the control valves whenever a disturbance in $\boldsymbol{d}$ or $\boldsymbol{r}$ takes place. This movement should satisfy certain performance criteria such as stability, robustness and disturbance attenuation.

The first question to be answered about the control system **K** is the pairing problem: since both inputs Q and G_u have influence on both outputs T and S, it has to be decided whether to control T with G_u or with Q. In the former choice S is controlled with Q, and in the second with G_u. Furthermore, if the "best" choice satisfies the stability criterion and produces a well-behaved response, then it follows that the inherent interaction of the systems was not too severe to prevent adequate performance with this simple diagonal control matrix. In order to resolve these questions, the interaction array (RGA) of Bristol [10] can be used:

$$\mathrm{B} = \mathrm{RGA} = \frac{(\partial Y_i)/(\partial U_j)_U}{(\partial Y_i)/(\partial U_j)_Y} \tag{15}$$

The RGA has gained wide acceptance in recent years and is now a standard calculation in process control systems. The numerator of eqn. (15) is the steady state gain between Y_i and U_j when all inputs but U_j are constant. The denominator is the same quantity but with the outputs except for Y_i held constant. The variables Y_i and U_j whose corresponding value of $\mathbf{B}_{ij}$ is closer to unity should be paired together. At least for a 2 × 2 system this choice, if available, means the following:

(i) The degrees of freedom assumed are confirmed.

(ii) A stable multivariable control system can be obtained.

(iii) The interacting control system can be effective with simple diagonal controllers, *i.e.* no further compensation is required.

These results are very significant since the RGA can be obtained easily from steady state information. If the steady state gain matrix denoted by $\mathbf{G}_{p(0)}$ is

$$\mathbf{G}_{p(0)} = \begin{bmatrix} K_{p\,11} & K_{p\,12} \\ K_{p\,21} & K_{p\,22} \end{bmatrix} \tag{16}$$

then

$$K_{p\,11} = \frac{(F5{*}F6{*}F11 + F11{*}F9{*}10)}{F7{*}F6} \tag{17a}$$

$$K_{p\,12} = \frac{(F11{*}F9)}{(F7{*}F6)} \tag{17b}$$

$$K_{p\,21} = F10/F6 \tag{17c}$$

$$K_{p\,22} = 1/F6 \tag{17d}$$

B can be calculated as

$$\mathbf{B} = [\mathbf{G}_{p(0)}][[\mathbf{G}_{p(0)}]^{-1}]^T \tag{18}$$

for a 2 × 2 system, this reduces to

$$\mathbf{B}_{11} = \frac{1}{1-(K_{p\,12}K_{p\,21})/(K_{p\,11}K_{p\,22})} \tag{19}$$

7. INTERACTING CONTROL OF AN ANAEROBIC DIGESTER

Calculation of the interaction array for both the thermophilic and the mesophilic case yields the following:

$$\mathbf{B}_T = \begin{array}{c} \begin{array}{cc} Q & \quad G_u \end{array} \\ \begin{bmatrix} -0.55 & 1.55 \\ 1.55 & -0.55 \end{bmatrix} \end{array} \begin{array}{c} T \\ S \end{array} \tag{20a}$$

$$\mathbf{B}_M = \begin{bmatrix} -0.2 & 1.2 \\ 1.2 & -0.2 \end{bmatrix} \tag{20b}$$

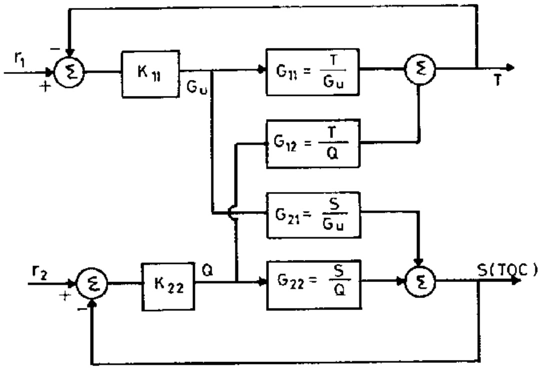

Fig. 10. Multivariable control of anaerobic digestion.

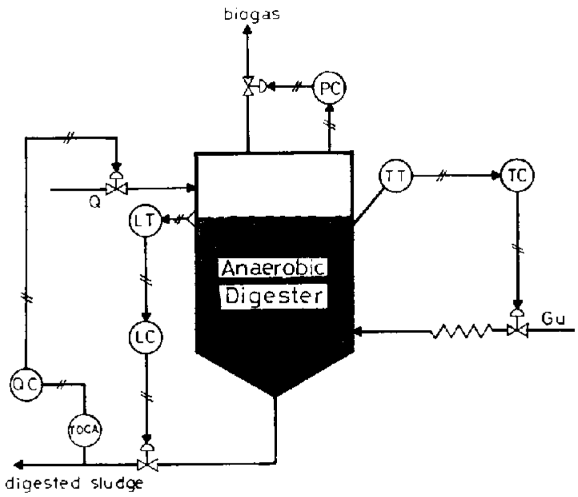

Fig. 11. Feedback control system for an anaerobic digester.

The terms closer to unity are those associated with the pairs T–G_u and S–Q. Thus this pairing is the recommended one. A block diagram of the resulting closed loop system is given in Fig. 10, where the columns of G_p are switched to account for the recommended permutation. A control concept diagram is shown in Fig. 11. The material balance control is also shown in Fig. 11 for gas and liquid inventory management. Details for designing the control matrix K are given by Alatiqi [11]. The structure of K, however, is already determined as diagonal, which is the simplest design for a multivariable system. It should be noted that the interaction characteristic for thermophilic operation is only slightly larger than that for mesophilic operation.

8. MORARI INTEGRAL CONTROLLABILITY (MIC) AND PROCESS RESILIENCE

Another useful feature of some multivariable control systems is integral controllability [12]. By MIC is meant that the system is stable under integral control action for some controller gains K_c greater than zero. This property is desirable for two reasons: first, most controllers involve integral action (in order to eliminate the steady state offset); secondly, this property allows controller tuning on-line by starting with a very small K_c and yet conserving stability. The condition for MIC is straightforward: the eigenvalues of $G^{+}_{p(0)}$ must lie in the open right-half complex plane. $G^{+}_{p(0)}$ is obtained from $G_{p(0)}$ by adjusting the sign of the columns such that the diagonal elements are positive. For the present process the eigenvalues of $G^{+}_{p(0)}$ are

$$\lambda_T = [0.365, 10.215]$$

$$\lambda_M = [1.033, 30.215]$$

So both systems are integral controllable. In order to compare the two systems for process resilience, it is noted that resilience is defined by the condition number of the appropriate process function [13] *i.e.* the condition number of G_p describes the resilience of the input–output pairs. The smaller the condition number $\gamma(G_p)$, the more resilient is the system. Mathematically, smaller values of $\gamma(G_p)$ indicate good conditioning of the system matrix and vice versa.

In process control $\gamma(G_p)$ is a measure of the sensitivity of the system to model uncertainty. The larger the value of γ, the more sensitive is the control performance to model uncertainty. The resiliency index γ was found to be 197 and 540 for the thermophilic and mesophilic cases respectively. Hence the thermophilic system is favoured. Another measure of process resilience is the inherent capacity of the system for disturbance attenuation. This capacity is measured in terms of a particular disturbance, and the representative measure is called the disturbance condition number γ_d, given by [14]

$$\gamma_d = \frac{\|G_p^{-1} G_d\|_2}{\|G_d\|_2} \sigma_{max}(G_p) \tag{21}$$

Again a smaller value of γ_d indicates favourable disturbance attenuation. It was

TABLE 3

Process gain matrices, singular values and condition numbers for the anaerobic digestion process

A. Mesophilic process

$$\mathbf{G}_{\mathrm{p}(0)} = \begin{bmatrix} 1.2866 & -128.3283 \\ -0.05 & 30 \end{bmatrix} \qquad \mathbf{G}_{\mathrm{d}(0)} = \begin{bmatrix} -4.2776 & 0.0123 \\ 1 & 0 \end{bmatrix}$$

$$\Sigma = \begin{bmatrix} 131.794 & 0 \\ 0 & 0.244 \end{bmatrix} \qquad \gamma_{\mathrm{M}} = 540$$

$\gamma_{\mathrm{d}1} = 0.94416$

$\gamma_{\mathrm{d}2} = 122.832$

B. Thermophilic process

$$\mathbf{G}_{\mathrm{p}(0)} = \begin{bmatrix} 0.5999 & -25.6812 \\ -0.0833 & 10 \end{bmatrix} \qquad \mathbf{G}_{\mathrm{d}(0)} = \begin{bmatrix} -2.5681 & 0.0133 \\ 1 & 0 \end{bmatrix}$$

$$\Sigma = \begin{bmatrix} 27.566 & 0 \\ 0 & 0.140 \end{bmatrix} \qquad \gamma_{\mathrm{T}} = 197$$

$\gamma_{\mathrm{d}1} = 0.9853$

$\gamma_{\mathrm{d}2} = 71.426$

found (Table 3) that thermophilic operation has a smaller value of γ_d with respect to organic load S_0 and hence is expected to have better disturbance attenuation characteristics. Both γ and γ_d are independent of the control algorithm used in the feedback line. This is also true for the relative gain array B. Hence this analysis, although steady state in nature, does give some insight to the relative closed-loop performance of candidate processes.

9. CONCLUSIONS

It has been shown from linear analysis of sludge anaerobic digestion that a multivariable control system can be designed. Digestion temperature and effluent concentration (measured in TOC) were taken as the quality control variables. Heat input and influent flow rate were chosen as the manipulative variables. Because of the interaction between these variables the pairing problem was addressed and it was found that a simple diagonal compensation could be designed by pairing T with G_u and S with Q. This control structure has been shown to be stabilizable and integral controllable.

Retention time and biogas pressure are controlled by effluent and biogas manipulation respectively. This control strategy, if adopted, would ensure the maintenance of the anaerobic digestion objectives: stabilization, pasteurization and steady biogas production.

Comparisons between thermophilic and mesophilic operation were carried out for relative gains, integral controllability, process resilience and disturbance attenuation. In none of these tests was it found that thermophilic operation was any less favourable than mesophilic operation. In fact, two of the tests (γ and γ_d) favoured the thermophilic process. Moreover, the shorter dynamic response of the thermophilic process would allow faster recovery from upsets.

These results suggest that under the proposed control scheme thermophilic digestion should be easier to control than mesophilic operation. This result is unexpected, since the common feeling reported in the literature [6, 7] suggests that elevated temperature operation poses a challenging control problem. However, most of these reports refer to the control problem from the point of view of the constraints posed by the smaller operating window of the thermophilic process rather than from detailed analysis of the control problem itself.

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APPENDIX A: LINEARIZED EQUATIONS FOR AN ANAEROBIC DIGESTER

$$\frac{\mathrm{d}S}{\mathrm{d}t} = \left(\frac{S_{0s} - S_s}{V}\right)Q + \left(\frac{Q_s}{V}\right)S_0 - \left(\frac{0.013 S_s X_s}{Y(K + S_s)}\right)T - \left(\frac{(0.013T_s - 0.129)S_s}{Y(K + S_s)}\right)$$

$$- \left(\frac{(0.013T_s - 0.129)X_s K}{Y(K + S_s)^2} + \frac{Q_s}{V}\right)S \tag{A1}$$

$$\frac{\mathrm{d}T}{\mathrm{d}t} = \left(\frac{Q_s}{V}\right)T_1 + \left(\frac{T_{1s} - T_s}{V}\right)Q - \left(\frac{Q_s}{V}\right)T + G_u \tag{A2}$$

APPENDIX B: NOMENCLATURE

A	state matrix
B	Bristol array (the relative gain array), or input matrix
C	control matrix
d	disturbance vector
G	biogas flow rate (m^3 day^{-1})
$\mathbf{G}_d$	disturbance transfer function
$\mathbf{G}_p$	process transfer function
G_u	specific heat addition rate (°C day^{-1})
HRT	hydraulic retention time
K, **K**	half-saturation constant (mg l^{-1}) or controllers matrix
K_{ii}	controller gain of the ith loop
K_{pij}	the steady state gain between input j and output i
Q	inflow rate (m^3 day^{-1})
S	effluent substrate concentration (mg (COD) l^{-1})
SRT	solid retention time
S_0	influent substrate concentration (mg (COD) l^{-1})
T	digestion temperature (°C)

T_0	influent temperature (°C)
t	time
U	input vector
V	volume of digester liquor (m^3)
X, $\mathbf{X}$	microorganism concentration (m^3 vss m^{-3}) or state vector
Y_g	gas yield rate (m^3 biogas (m^3 vss)$^{-1}$ day^{-1})
Y, $\mathbf{Y}$	substrate yield rate (m^3 l^{-1} (m^3 m^{-3})$^{-1}$) or output vector

Greek symbols

Γ	disturbance matrix
γ	resiliency index, the condition number of $\mathbf{G}_{p(0)}$
γ_d	disturbance condition number
λ	eigenvalue
μ	specific growth rate day^{-1}
μ_{max}	maximum specific growth rate day^{-1}
Σ	diagonal matrix containing singular values σ_{ii}

Subscript

s	denotes steady state values