Comparison between dynamics and control performance of mesophilic and thermophilic anaerobic sludge digesters

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Abstract

Two anaerobic digestion schemes, namely mesophilic and thermophilic processes, have been analysed for process stability and controllability. As in any control problem, three types of variable are distinguished in this study, *i.e.* disturbance, manipulated and controlled variables. Substrate concentration S_i and influent temperature T_i are the disturbance variables. The manipulated variables are sewage sludge influent rate Q and specific heat addition rate G_u. The controlled variables are effluent substrate concentration S and digestion temperature T. A control system including a proportional–integral (PI) controller and variable groups is proposed. Multivariable steady state methods such as relative gain array, Niederlinski stability criterion, singular value decomposition and Morari integral controllability are employed. Several dynamic analyses such as biggest log-modulus tuning, robustness analysis and Tyreus load rejection criterion are carried out. Steady state analysis results are used for variable pairing. The results of multivariable analysis show that thermophilic anaerobic digestion is more favourable in terms of speed of response and disinfection capability. It also maintains dynamic and steady state stability if controlled properly.

1. Introduction

A generalized anaerobic system can be visualized as a physicochemical system interacting with a biochemical system. The physicochemical system consists of gas, liquid and a biologically inert fraction of solids. The biochemical system consists of microbial cells and related exoenzymes which act as a mass and energy transfer unit. Interactions between the physicochemical and biochemical processes are complicated owing to strong influence of carbon dioxide or carbonic acid equilibria and high values of carbon dioxide partial pressure. Anaerobic degradation of compounds to carbon dioxide and water is carried out in series by various microbial populations each feeding on metabolites produced by the organisms. Anaerobic processes are used to digest waste waters high in suspended solids. The particulate suspended solids entering a biological reactor affect the composition of the mixed liquor sludge with respect to the fraction of active biomass. Thus anaerobic processes are inherently complex and can be described only with many assumptions [1, 2].

The slow growth of methanogenic bacteria such as methanobacterium, methanococcus, methanosarcina and methanospirillum adds a few disadvantages to the process, *e.g.* (i) time-consuming laboratory experiments, causing time factor limitations, to comprehend the dynamic behaviour of an anaerobic reactor and (ii) poor process stability. By designing a good control system, these can be overcome. Suitable dynamic mathematical models could be useful to evaluate the process behaviour and to formulate and evaluate digester control strategies [3–5]. From the results of Alatiqi *et al.* [6] it is suggested that substrate concentration S and digestion temperature T are selected as the two controlled variables and inflow rate Q and heat input G_u are taken as manipulated variables. It is also clear from their study that inlet temperature T_i and inlet substrate concentration S_i are the two major disturbances of the system. Various methods such as relative gain array (RGA), Niederlinski stability criterion (Niederlinski index, NI), singular value decomposition (SVD) and Morari integral controllability (MIC) are employed to check the interaction analysis and integral controllability of the mesophilic and thermophilic processes. Tuning and detuning of the controller to meet the stability and load rejection criteria were also carried out by methods

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such as biggest log-modulus tuning (BLT), robustness analysis (RA) and Tyreus load rejection criterion (TLC).

The present work was carried out with the objectives of (i) designing a closed-loop control scheme for an anaerobic digester operating in the mesophilic and thermophilic ranges of temperature (for this purpose the method proposed by Yu and Luyben [7] is used) and (ii) analysing the controllability and closed-loop response of the anaerobic digestion process.

Lastly, the performance of thermophilic and mesophilic operations will be compared using the above methods and dynamic simulation.

2. Dynamic model

The digestion model can be presented in state space form using the equations given below and expanded in Appendix A.

If $\boldsymbol{A}$ is the coefficient matrix of the controlled variables $\boldsymbol{X}$, $\boldsymbol{B}$ is the coefficient matrix of the manipulated variables $\boldsymbol{U}$ and $\boldsymbol{\Gamma}$ is the coefficient matrix of the disturbance $\boldsymbol{d}$, one obtains

$$\boldsymbol{X}=\boldsymbol{AX}+\boldsymbol{BU}+\boldsymbol{\Gamma d} \tag{1}$$

where the vectors $\boldsymbol{X}$, $\boldsymbol{U}$ and $\boldsymbol{d}$ are given by

$$\boldsymbol{X}=\begin{bmatrix} S \\ T \end{bmatrix}, \quad \boldsymbol{U}=\begin{bmatrix} Q \\ G_u \end{bmatrix}, \quad \boldsymbol{d}=\begin{bmatrix} T_o \\ S_o \end{bmatrix}$$

The symbols S, T, Q, G_u, T_o and S_o represent the effluent substrate concentration, digestion temperature, influent flow rate, specific heat addition rate, steady state temperature and steady state concentration respectively.

Equation (1) can be written as

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{Bmatrix} X \\ S \\ T \end{Bmatrix}=\begin{bmatrix} 0 & F_3 & F_1 \\ -F_8 & -F_7 & -F_9 \\ 0 & 0 & -F_6 \end{bmatrix}\begin{Bmatrix} X \\ S \\ T \end{Bmatrix} + \begin{bmatrix} -F_4 & 0 \\ F_5 & 0 \\ F_{10} & 1 \end{bmatrix}\begin{Bmatrix} Q \\ G_u \end{Bmatrix} + \begin{bmatrix} 0 & 0 \\ F_6 & 0 \\ 0 & 0 \end{bmatrix}\begin{Bmatrix} S_i \\ T_i \end{Bmatrix} \tag{2}$$

where S_i and T_i are the influent substrate concentration and temperature respectively and F_3, F_1, F_4, F_8, F_7, F_9, F_5, F_6 and F_{10} are functions of the process parameters as defined in Table 1.

If $\boldsymbol{C}$ is a measurement transformation matrix that converts the measured states $\boldsymbol{X}$ to the output $\boldsymbol{Y}$, then the relation between vectors and $\boldsymbol{Y}$ and $\boldsymbol{X}$ can be given as

$$\boldsymbol{Y}=\boldsymbol{CX} \tag{3}$$

TABLE 1. Functions of process parameters

$$F_1=\frac{0.013S_sX_s}{K+S_s}$$

$$F_2=\frac{(0.013T_s-0.129)S_s}{(K+S_s)-Q_s/V}$$

$$F_3=\frac{(0.013T_s-0.129)X_sK}{(K+S_s)^2}$$

$$F_4=\frac{-X_s}{V}$$

$$F_5=\frac{S_{0s}-S_s}{V}$$

$$F_6=\frac{Q_s}{V}$$

$$F_7=\frac{-(0.013T_s-0.129)KX_s}{YX(K+S_s)^2-Q_s/X}$$

$$F_8=\frac{-(0.013T_s-0.129)S_s}{YX(K+S_s)-Q_s/V}$$

$$F_9=\frac{-0.013S_sX_s}{YX(K+S_s)}$$

$$F_{10}=\frac{T_{0s}-T_s}{V}$$

where

$$\boldsymbol{Y}=\begin{bmatrix} \mathrm{TOC} \\ T \end{bmatrix}$$

Our model is based on the measurement of substrate concentration S using the chemical oxygen demand (COD) method. The recommended real time measurement of S is total organic carbon (TOC). From correlations in the literature COD is 2.2 times TOC [8].

On setting $F_{11}=1/2.2$, then

$$\boldsymbol{C}=\begin{bmatrix} 0 & F_{11} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Figures 1–4 show the effect of inputs on the substrate concentration and Figs. 5 and 6 show the response of the micro-organism concentration to Q and S_i. From Fig. 3 it is seen that the substrate concentration increases with increasing inlet substrate concentration, but reaches a steady state value with increasing micro-organism concentration as seen in Fig. 6. Thus the figures clearly show that the thermophilic case is more sensitive and faster in responding to input perturbations.

3. Open-loop transfer functions

The process and disturbance gain matrices $G_{p(s)}$ and $G_{d(s)}$ are obtained by performing the Laplace transform of eqns. (1) and (3). Thus

$$G_{p(s)} = \begin{bmatrix} \dfrac{F_5F_{11}S^2+(F_5F_{11}F_6+F_4F_{11}F_8-F_{11}F_9F_{10})S+(F_1F_{11}F_8F_6-F_1F_8F_{10}F_{11})}{(S+F_6)(S^2+F_7S+F_3F_8)} & \dfrac{-F_{11}F_9S-F_1F_8F_{11}}{(S+F_6)(S^2+F_7S+F_3F_8)} \\ \dfrac{F_{10}}{S+F_6} & \dfrac{1}{S+F_6} \end{bmatrix} \tag{4}$$

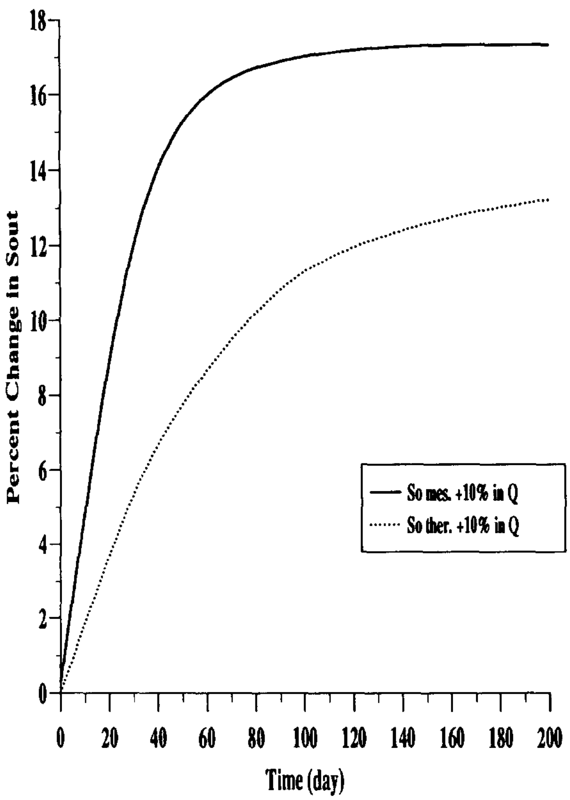

Fig. 1. Response of substrate concentration to +10% perturbation in Q.

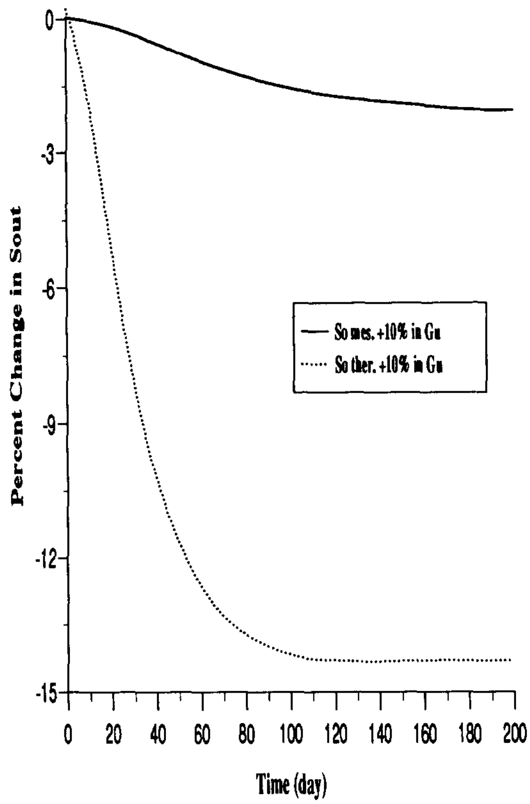

Fig. 2. Response of substrate concentration to +10% perturbation in G_u.

$$G_{d(s)} = \begin{bmatrix} \dfrac{F_{11}F_6S}{S^2+F_7S+F_3F_8} & \dfrac{-F_{11}F_9F_6S-F_1F_8F_{11}F_6}{(S+F_6)(S^2+F_7+F_3F_8)} \\ 0 & \dfrac{F_6}{S+F_6} \end{bmatrix} \tag{5}$$

By solving the polynomials in the numerators and denominators with numerical values and after simplification, the process and disturbance transfer function matrices for the mesophilic and thermophilic cases were obtained as given in Appendices B and C.

4. Steady state analysis techniques and variable pairing

When the off-diagonal elements in the transfer function matrix are not equal to zero, then there is some sort of interaction between the inputs and outputs in the system. As seen in the transfer function matrix, the off-diagonal elements are not equal to zero. Thus there is some level of interaction between the inputs Q and G_u and outputs S and T. An open-loop block diagram for this case is shown in Fig. 7.

Some of the steady state analysis techniques that are employed here to test the proposed control

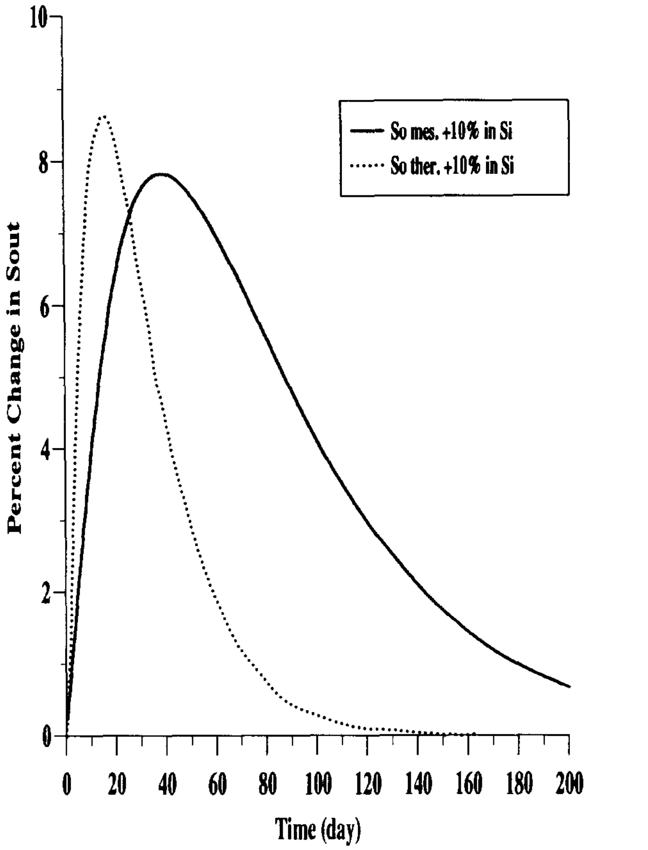

Fig. 3. Response of substrate concentration to +10% perturbation in S_i.

scheme for both the thermophilic and mesophilic cases are (i) relative gain array [8, 9], (ii) Niederlinski stability criterion [10], (iii) singular value decomposition [11] and (iv) Morari integral controllability [9].

Because of the scale sensitivity of steady state analysis tools, the transfer functions are made dimensionless by including the sensor and actuator gains as follows:

$$G_{pc}=\begin{bmatrix} G_{m1}G_{p11}G_{v1} & G_{m1}G_{p12}G_{v2} \\ G_{m2}G_{p21}G_{v1} & G_{m2}G_{p22}G_{v2} \end{bmatrix}$$

The following assumptions were made for measuring devices and valve gains.

(1) The signal span is assumed to be 10–50 mA.

(2) For the mesophilic case the steady state capacity is assumed to be doubled and the range of flow rate is taken from 0 to 540 m^3 day^{-1}.

(3) For TOC analysis a measurement range from 6000 to 10 000 mgTOC l^{-1} is assumed.

(4) The fuel valve range of G_u is assumed to be between 0 and 0.6 °C day^{-1} and the thermocouple temperature range is taken to be from 10 to 60 °C.

(5) The thermocouple sensing range is assumed to be between 10 and 70 °C.

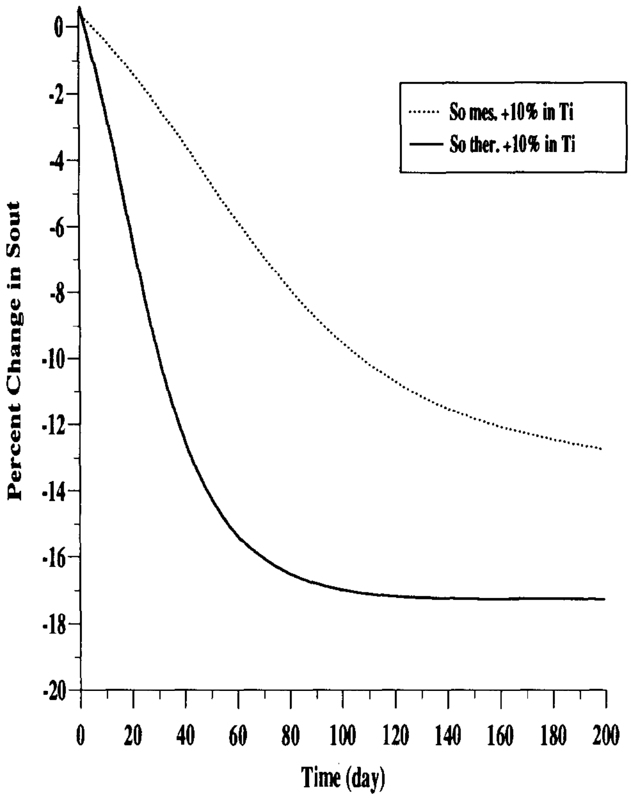

Fig. 4. Response of substrate concentration to +10% perturbation in T_i.

The sensor and actuator gains based on the above assumptions are given in Table 2.

Thus

$$\begin{array}{c} \quad\quad Q \quad\quad\quad G_u \\ G_{m(0)}=0\begin{bmatrix} 5.712 & -1.709 \\ -0.2 & 0.36 \end{bmatrix}\begin{array}{c} S \\ T \end{array} \end{array}$$

and

$$\begin{array}{c} \quad\quad Q \quad\quad\quad G_u \\ G_{t(0)}=\begin{bmatrix} 8.014 & -3.431 \\ -0.833 & 1 \end{bmatrix}\begin{array}{c} S \\ T \end{array} \end{array}$$

where subscripts "m" and "t" represent the mesophilic and thermophilic processes respectively. The block diagram of the closed-loop system is shown in Fig. 8 and the digester control system is shown in Fig. 9.

4.1. Relative gain array (RGA)

One of the well-known measures for interaction analysis is the RGA [8], which works on the steady state transfer function $G_{(0)}$. Knowing $\boldsymbol{G}_{(0)}$, an estimation of the $\boldsymbol{B}$ matrix is made. Here an element B_{ij} of the $\boldsymbol{B}$ matrix represents the interaction between input i and output j.

Grosdidier *et al.* [9] predicted the B_{ij} element of the RGA to be

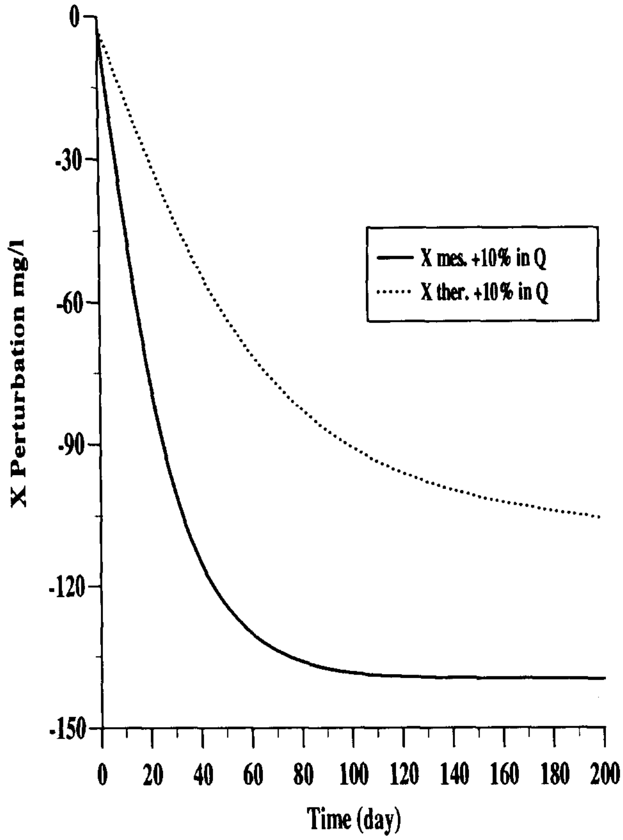

Fig. 5. Response of micro-organism concentration to +10% perturbation in Q.

$$B_{ij}=K_{ij}(K_{ij}-1) \quad (6)$$

where i and j denote input and output values respectively and K_{ij} and $K_{ij}-1$ are elements of $\boldsymbol{G}_{(0)}$ and $\boldsymbol{G}_{(0)}-1$ respectively. The sum of the elements in any row or column of $\boldsymbol{B}$ is equal to unity; when one element is equal to unity, this means that the other loops have no effect on this loop and hence are free from interaction. Therefore the criteria for pairing inputs and outputs using the RGA method are to choose pairings that give B_{ij} which are closer to unity and avoid pairings with negative relative gain values.

Thus

$$\boldsymbol{B}_{\rm m}=\begin{array}{c} \begin{array}{cc} Q & G_{\rm u} \end{array} \\ \left[\begin{array}{cc} 1.2 & -0.2 \\ -0.2 & 1.2 \end{array}\right] \end{array} \begin{array}{c} \\ S \\ T \end{array}$$

and

$$\boldsymbol{B}_{\rm t}=\begin{array}{c} \begin{array}{cc} Q & G_{\rm u} \end{array} \\ \left[\begin{array}{cc} 1.55 & -0.55 \\ -0.55 & 1.55 \end{array}\right] \end{array} \begin{array}{c} \\ S \\ T \end{array}$$

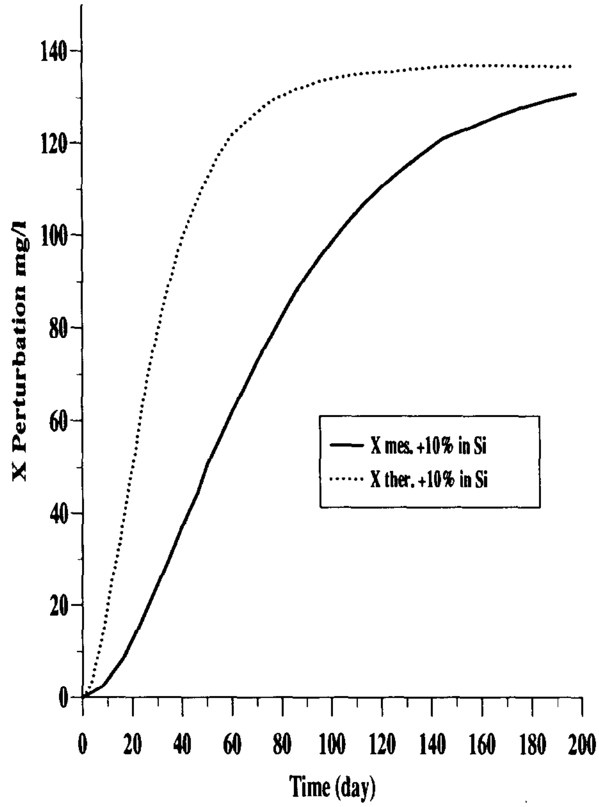

Fig. 6. Response of micro-organism concentration to +10% perturbation in $S_{\rm i}$.

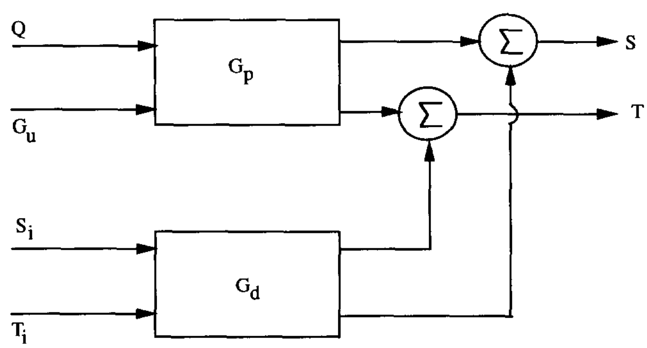

Fig. 7. Open-loop diagram of anaerobic digester.

TABLE 2. Sensor and actuator gains

Gain		Unit	Mesophilic	Thermophilic
G_{v1}	Q	m^3 day^{-1} mA^{-1}	13.5	40.5
G_{m1}	S	mA l $mgTOC^{-1}$	0.01	0.01
G_{v2}	G_u	°C day^{-1} mA^{-1}	0.015	0.15
G_{m2}	T	mA $°C^{-1}$	0.8	0.667

Here the values of $\boldsymbol{B}_{\rm m}$ and $\boldsymbol{B}_{\rm t}$ indicate that the diagonal elements are those which are closer to unity. Thus S–Q and T–$G_{\rm u}$ are the recommended pairings.

In both cases the interaction is small and no interaction compensator or decoupler is required.

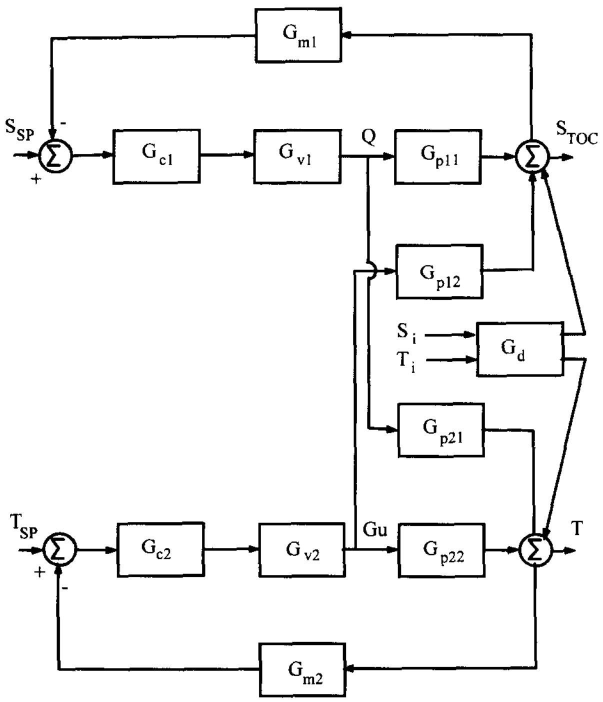

Fig. 8. Block diagram of closed-loop system for anaerobic digester.

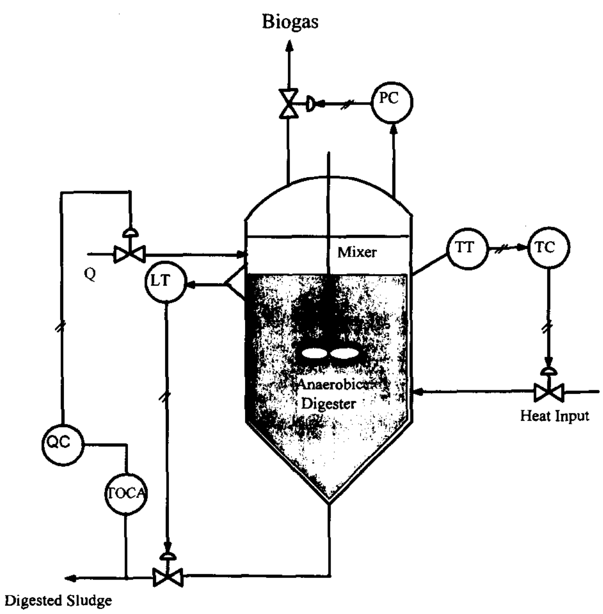

Fig. 9. Feedback control system for anaerobic digester.

4.2. Niederlinski stability criterion

The Niederlinski index (NI) is defined as

$$\mathrm{NI}=\frac{\det[\boldsymbol{G}_{(0)}]}{\prod_{i=1}^{n} k_{ii}} \tag{7}$$

The Niederlinski index for the mesophilic and thermophilic schemes is calculated from equation (7). Positive values of $\mathrm{NI}_m=0.833$ and $\mathrm{NI}_t=0.643$ are obtained, indicating the proposed control scheme to be stable.

4.3. Singular value decomposition (SVD)

SVD is a method to decompose a complex $n\times n$ matrix $\boldsymbol{A}$ into three component matrices according to

$$\boldsymbol{A}=\boldsymbol{U\Sigma V}^T$$

where $\boldsymbol{U}$ and $\boldsymbol{V}$ are unitary matrices ($\boldsymbol{U}^T=\boldsymbol{U}^{-1}$) and $\boldsymbol{\Sigma}$ is a diagonal matrix of singular values (σ_is). The ratio $\bar{\sigma}/\underline{\sigma}$, where $\bar{\sigma}$ and $\underline{\sigma}$ are the maximum and minimum singular values of $G_{p(0)}$, is defined as the plant condition number (CN). A large condition number indicates a large degree of directionality (degree of dependence of variables or interaction). The singular value analysis results for the mesophilic case are

$$\sigma_i=5.97,\quad 0.287$$

$$\boldsymbol{u}=\begin{bmatrix}0.999 & -4.95\times10^{-2}\\ -4.95\times10^{-2} & -0.999\end{bmatrix}$$

$$\boldsymbol{v}=\begin{bmatrix}0.957 & -0.289\\ -0.289 & -0.957\end{bmatrix}$$

$$\mathrm{CN}=20.7$$

and for the thermophilic case

$$\sigma_i=8.795,\quad 0.586$$

$$\boldsymbol{v}=\begin{bmatrix}0.915 & -0.402\\ -0.402 & -0.915\end{bmatrix}$$

$$\mathrm{CN}=15$$

These results show that the interaction is higher in the mesophilic case. Therefore the condition number for the thermophilic case is lower, meaning that the interaction is lower in the thermophilic case than in the mesophilic one. This is in contrast with the RGA results.

4.4. Morari integral controllability (MIC)

A system is defined to be integrally controllable if there exists a range of positive gains starting from zero for which the closed-loop system is stable. A sufficient and necessary condition for integral controllability is that all eigenvalues of the $\boldsymbol{G}^+_{(0)}$ matrix be positive. $\boldsymbol{G}^+_{(0)}$ is the plant steady state gain matrix with signs adjusted so that all diagonal elements have positive signs. For the digester model, in both the mesophilic and thermophilic cases, the diagonal elements of $\boldsymbol{G}^+_{(0)}$ are positive, so that sign adjustment is not needed. The MIC results for the mesophilic and thermophilic cases respectively are

$$\lambda_m=5.775,\quad 0.297$$

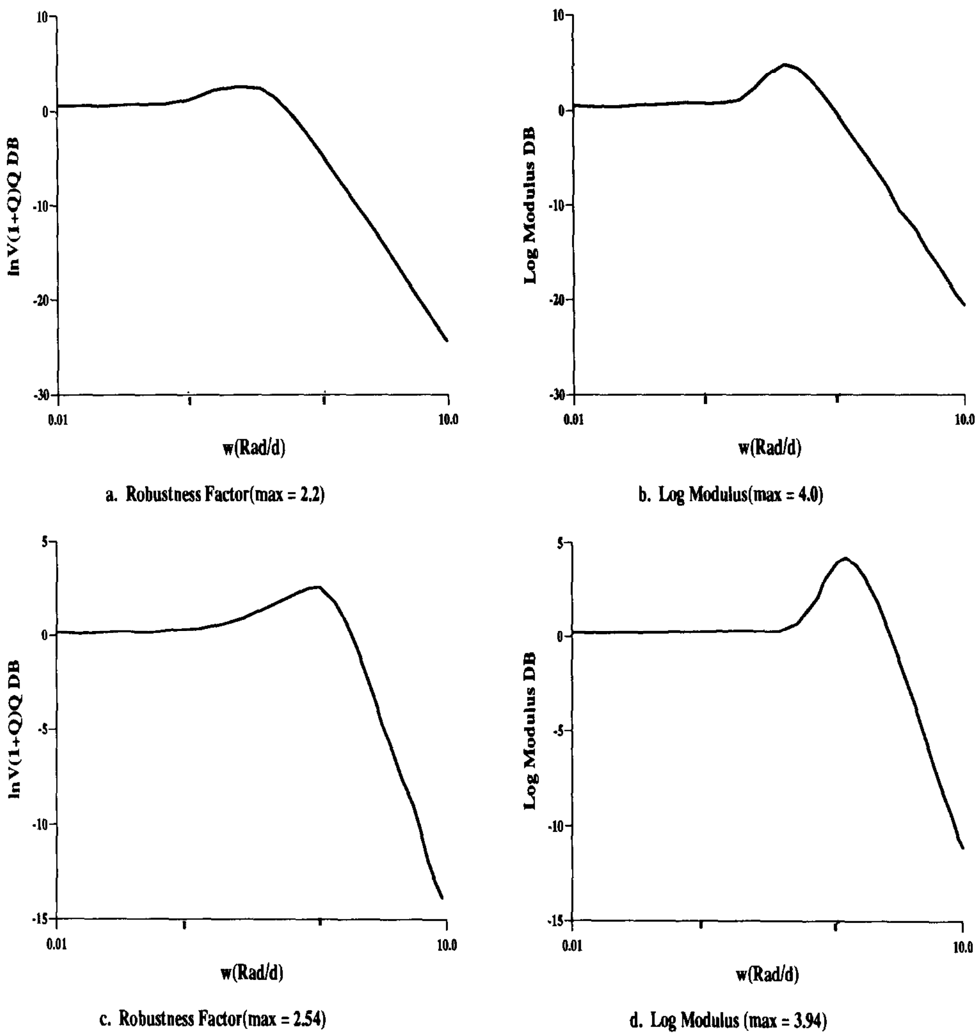

Fig. 10. Control system analysis in frequency domain. (a), (b) Mesophilic case; (c), (d) thermophilic case.

$\lambda_t = 8.4, \quad 0.614$

where λ denotes an eigenvalue. All λs are positive, so that the MIC criterion is met in both cases.

The MIC analysis results show that both systems are integrally controllable. Thus by controlling the system with PI-type controllers, there are some values of K_c which make the system stable.

5. Multivariable frequency response and dynamic analysis

The closed-loop system can be written as

$$S_{(s)} = P_{11} S_{sp(S)} + P_{12} T_{sp(S)} + D_{11} S_{i(S)} + D_{12} T_{i(S)} \tag{8}$$

$$T_{(s)} = P_{21} S_{sp(S)} + P_{22} T_{sp(S)} + D_{21} S_{i(S)} + D_{22} T_{i(S)} \tag{9}$$

where

$$P_{11} = \frac{G_{p11} G_{c1} G_{v1} + G_{c1} G_{v1} G_{c2} G_{v2} G_{m2}(G_{p11} G_{p22} - G_{p12} G_{p21})}{DN}$$

$$P_{12} = \frac{G_{p12} G_{c2} G_{v2}}{DN}$$

$$D_{11} = \frac{G_{d11} G_{c2} G_{v2} G_{m2}(G_{d11} G_{p22} - G_{d21} G_{p12})}{DN}$$

$$D_{12} = \frac{G_{d12} G_{c2} G_{v2} G_{m2}(G_{d12} G_{p22} - G_{d22} G_{p12})}{DN}$$

$$P_{21} = \frac{G_{p21} G_{c1} G_{v1}}{DN}$$

$$P_{22} = \frac{G_{p22} G_{c2} G_{v2} + G_{c1} G_{v1} G_{m1} G_{c2} G_{v2}(G_{p11} G_{p22} - G_{p12} G_{p21})}{DN}$$

$$D_{21} = \frac{G_{d21} G_{c1} G_{v1} G_{m1}(G_{p11} G_{d21} - G_{p21} G_{d11})}{DN}$$

$$D_{22} = \frac{G_{d22} G_{c1} G_{v1} G_{m1}(G_{p11} G_{d21} - G_{p21} G_{d11})}{DN}$$

and

$$DN = (1 + G_{p11} G_{c1} G_{v1} G_{m1})(1 + G_{p22} G_{c2} G_{v2} G_{m2})$$

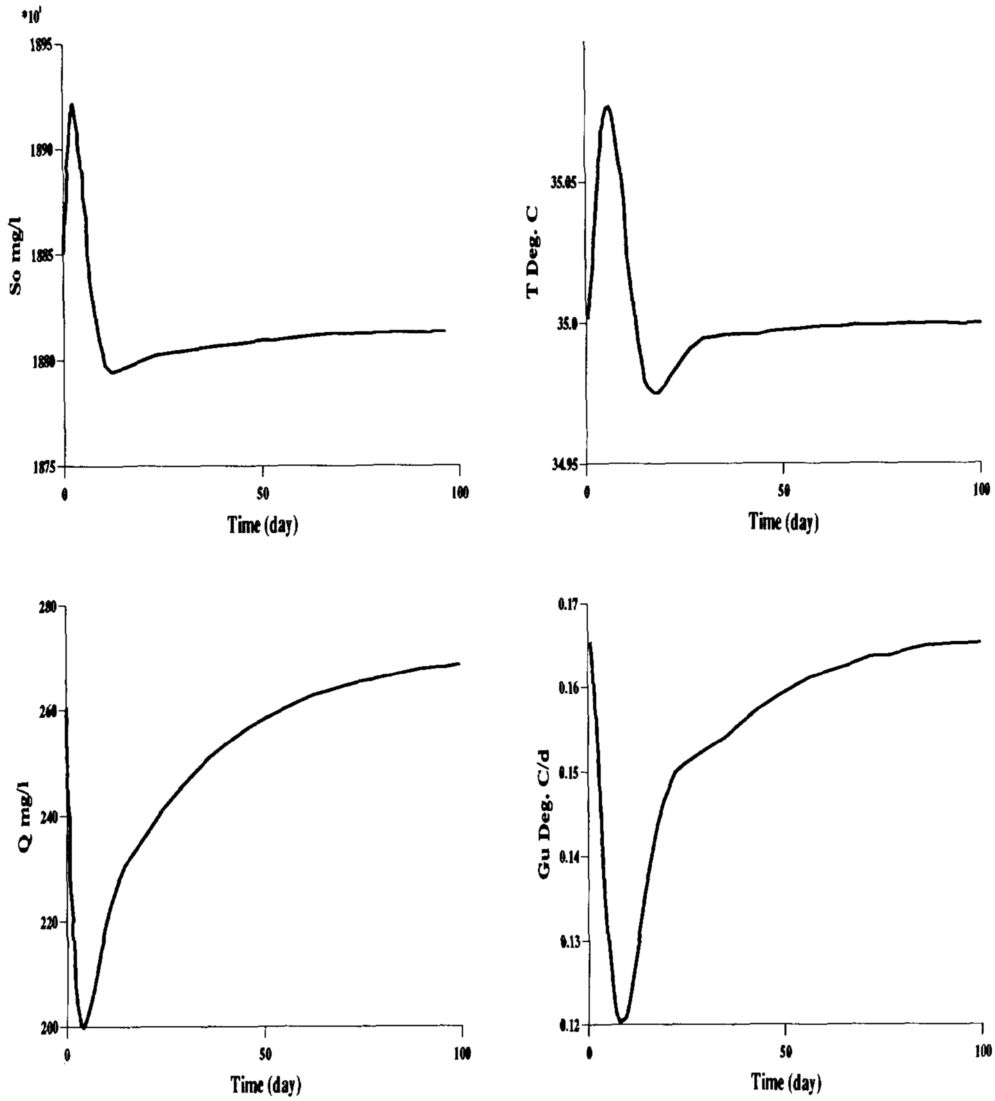

Fig. 11. Response of linearized mesophilic system to +10% step change in S_i load.

$$-G_{p21}G_{c1}G_{v1}G_{m1}G_{p12}G_{c2}G_{v2}G_{m2}$$

For tuning the two multiloop controllers and analysing the closed-loop performance, the following methods are used here: (i) biggest log-modulus tuning (BLT) [12]; (ii) robustness analysis (RA) [13]; (iii) Tyreus load rejection criterion (TLC) [14].

5.1. Biggest log-modulus tuning (BLT)

This method, proposed by Luyben [12], uses a multivariable Nyquist plot and equal detuning of all loops from the single-loop Ziegler–Nichols (ZN) setting.

The log-modulus L_c is calculated as

$$L_{c(iw)}=20\log\left|\frac{W}{1+W}\right| \tag{10}$$

where

$$W_{(iw)}=-1+\det(\boldsymbol{I}+\boldsymbol{G}_{(iw)}\boldsymbol{C}_{(iw)}) \tag{11}$$

Here $\boldsymbol{C}$ is the diagonal matrix of single-input–single-output (SISO) feedback controllers.

For the mesophilic case two PI controllers were tested for the BLT criterion. It was found that values of $K_{c1}=6$, $K_{c2}=25$, $\tau_{i1}=3.5$ days and $\tau_{i2}=4$ days meet the BLT criterion.

5.2. Robustness analysis

The ability of a system to remain stable over a range of parameter changes due to model uncertainty is called robustness. Seldom is the model identical with the real process. Tyreus [13] gave a measure for the robustness of a control system as

$$\|\Delta G\|<\frac{1}{\bar{\sigma}(I+Q)^{-1}Q} \tag{12}$$

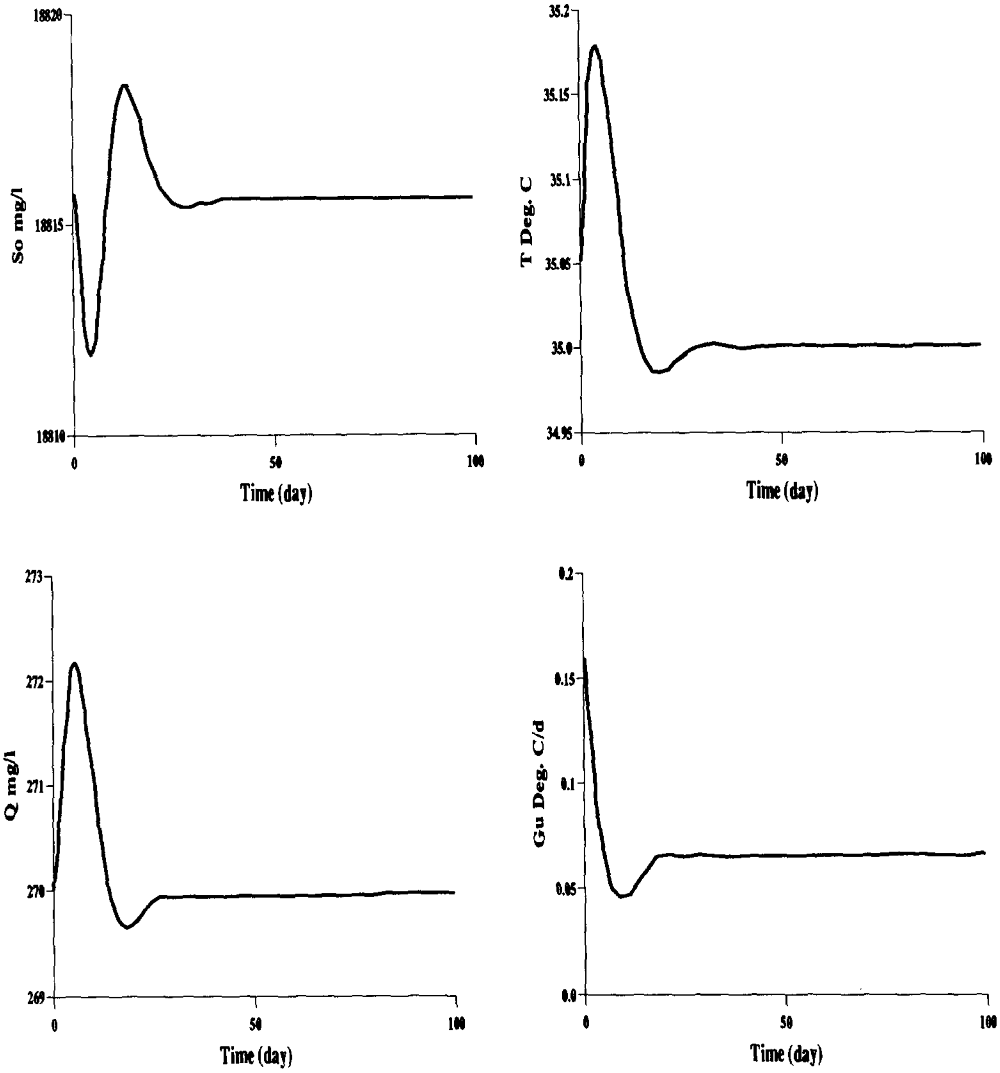

Fig. 12. Response of linearized mesophilic system to +10% step change in T_i load.

where ΔG is the model uncertainty and $\bar{\sigma}(I+Q)^{-1}Q$ is called the robustness factor.

It is suggested that the control system should be tuned such that the maximum robustness factor is less than 20 dB, giving a control system with a ΔG of 10% or less.

For the mesophilic case various values of gains and reset times of two PI controllers were tested for the BLT criterion. It was found that the values given above for the BLT criterion meet the robustness criterion. Figure 10(a) shows the robustness factor with a maximum of 2.2 dB. Figure 10(b) shows the log-modulus curve with the proposed controller settings. $L_{c_{max}}$ is equal to 4 dB. This is plotted according to eqn. (10). A similar procedure is applied to the thermophilic case and suitable controller gains and reset times found to be $K_{c1}=6$, $K_{c2}=9$, $\tau_{i1}=1.5$ days and $\tau_{i2}=17$ days. Robustness factor and log-modulus curves are plotted in Figs. 10(c) and 10(d) respectively. The maximum robustness factor in this case is 2.54 dB and $L_{c_{max}}$ is equal to 3.94 dB.

5.3. Closed-loop response

The time responses of the linear mesophilic model to +10% step changes in the loads S_i and T_i are shown in Figs. 11 and 12 respectively. The corresponding responses for the thermophilic case are shown in Figs. 13 and 14.

The curves show significant differences between the two operating conditions in terms of response time. The mesophilic operation settles in 30 days while the thermophilic operation settles in 10 days. Set point responses are similar in this regard. The order of magnitude of peak overshoot or maximum swing is similar. This result is very significant in many aspects. It confirms the predictions of the

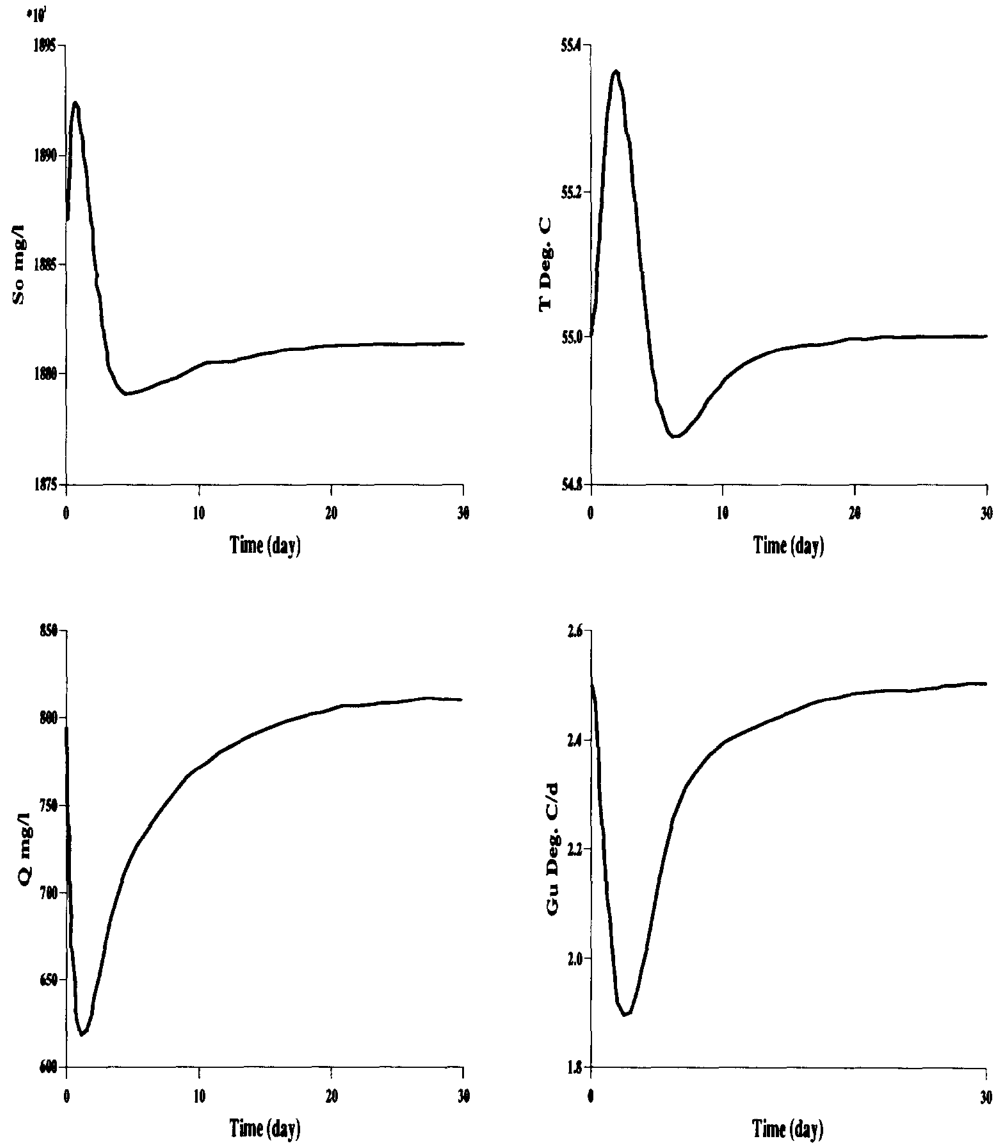

Fig. 13. Response of linearized thermophilic system to +10% step change in S_i load.

steady state analysis in that interaction levels are comparable between the two operations. Further, it shows that the thermophilic operation, if controlled properly, is no more difficult to control than the mesophilic one. In fact, the faster recovery following upsets would make the thermophilic operation more attractive and less confusing to operators following organic shocks. Such shocks are typically recurrent every weekend in Kuwait farming communities, which causes disturbances to the mesophilic operation of higher frequency than its closed-loop time constant.

6. Conclusions

An effort was made to design a closed-loop control scheme for an anaerobic process. The interaction analysis and integral controllability of the process were carried out using methods such as relative gain array, Niederlinski stability criterion, singular value decomposition and Morari integral controllability.

The steady state analysis shows that although both systems are stable, the level of interaction is not the same in them. The RGA analysis shows that the thermophilic digestion has more interaction, while the SVD results show that the interaction is higher in the mesophilic case. However, the results in both cases are so close to each other that some scaling changes may reverse them. Owing to the low interaction values, it was decided that multiloop controllers are sufficient to control the process efficiently.

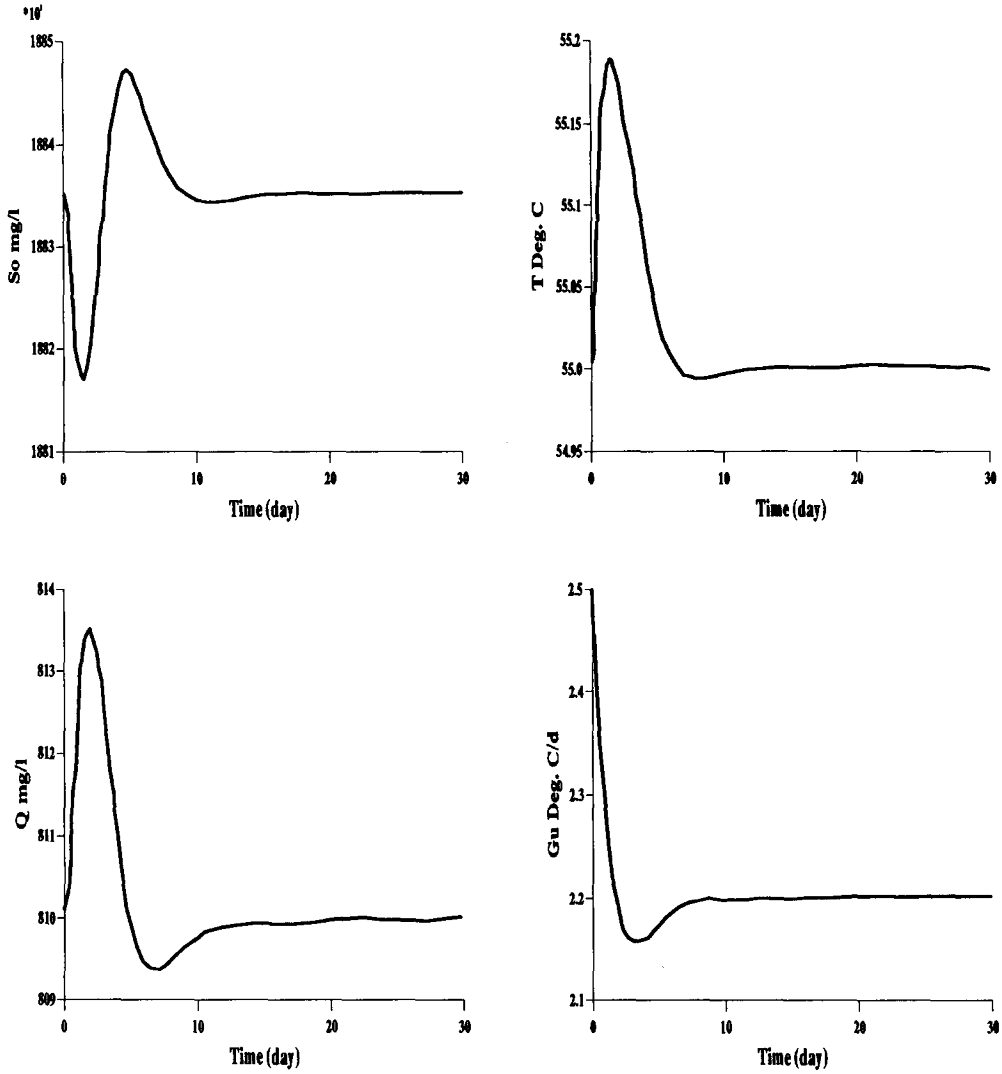

Fig. 14. Response of linearized thermophilic system to +10% step change in T_i load.

Tuning and detuning of the controllers to meet the stability and load rejection criteria were carried out using methods such as biggest log-modulus tuning and robustness analysis. From these methods it was concluded that both processes are stable and robust with BLT settings.

The time response curves show that the mesophilic operation is three times slower than the thermophilic one. The dynamic error is comparable between the two modes. The thermophilic operation maintains the disinfection advantage but requires more investment for heating.

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Appendix A: Linerarized equations for an anaerobic digester

$$\frac{dS}{dt}=\left(\frac{S_{i_s}-S_s}{V}\right)Q+\left(\frac{Q_s}{V}\right)S_i-\left(\frac{0.013S_sX_s}{Y_x(K_s+S_s)}\right)T-\left(\frac{(0.013T_s-0.129)S_s}{Y_x(K_s+S_s)}\right)X-\left(\frac{(0.013T_s-0.129)X_sK_s}{Y_x(K_s+S_s)^2}+\frac{Q_s}{V}\right)S \quad (A1)$$

$$\frac{dT}{dt}=\left(\frac{Q_s}{V}\right)T_i+\left(\frac{T_{i_s}-T_s}{V}\right)Q-\left(\frac{Q_s}{V}\right)T+G_u \quad (A2)$$

$$\frac{dX}{dt}=\left(\frac{0.013S_sX_s}{K_s+S_s}\right)T+\left(\frac{(0.013T_s-0.129)S_s}{K_s+S_s}-\frac{Q_s}{V}\right)X+\left(\frac{(0.013T_s-0.129)X_sK_s}{(K_s+S_s)^2}\right)S-\left(\frac{X_s}{V}\right)Q \quad (A3)$$

Appendix B: Transfer functions for mesophilic case

$$G_{p11}=\frac{42.312(25.013S+1)}{(30S+1)(49.237S+1)}$$

$$G_{p12}=\frac{-11395.022}{(30S+1)(49.237S+1)}$$

$$G_{p21}=\frac{-1.851\times10^{-2}}{30S+1}$$

$$G_{p22}=\frac{30}{30S+1}$$

$$G_{d11}=\frac{22.385}{(30S+1)(49.237S+1)}$$

$$G_{d12}=\frac{-379.834}{(30S+1)(49.237S+1)}$$

$$G_{d21}=0$$

$$G_{d22}=\frac{1}{30S+1}$$

Appendix C: Transfer functions for thermophilic case

$$G_{p11}=\frac{19.789(6.432S+1)}{(10S+1)(17.766S+1)}$$

$$G_{p12}=\frac{-2287.341}{(10S+1)(17.766S+1)}$$

$$G_{p21}=\frac{-3.086\times10^{-2}}{10S+1}$$

$$G_{p22}=\frac{10}{10S+1}$$

$$G_{d11}=\frac{8.075S}{(10S+1)(17.766S+1)}$$

$$G_{d12}=\frac{-228.734}{(10S+1)(17.766S+1)}$$

$$G_{d21}=0$$

$$G_{d22}=\frac{1}{10S+1}$$

Appendix D: Nomenclature

- G_u specific heat addition rate (°C day^{-1})
- K_s half-saturation constant (mg l^{-1})
- Q influent flow rate (m^3 day^{-1})
- Q_s saturated influent flow rate (m^3 day^{-1})
- S effluent substrate concentration (mgCOD l^{-1})
- S_i influent substrate concentration (mgCOD l^{-1})
- S_s saturated substrate concentration (mgCOD l^{-1})
- T digestion temperature (°C)
- T_i influent temperature (°C)
- T_s saturated digestion temperature (°C)
- V volume of digester liquor (m^3)
- X micro-organism concentration (mg l^{-1})
- X_s saturated micro-organism concentration (mg l^{-1})
- Y_x micro-organism yield rate X/S (mg mgCOD^{-1})